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Epistemological Analysis of Multiple Representations in Secondary Mathematics: The Case of the Function Concept

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***Abstract*— *Mathematical concepts are inherently inaccessible to learners except through their representations. Yet what does it mean to "represent" a mathematical object, and why does the movement between different forms of representation constitute such a persistent site of difficulty? This paper presents a sustained theoretical analysis that integrates Duval's theory of semiotic representation registers with Bachelard's notion of epistemological obstacles, using Sfard's reification framework as a bridging construct. Taking the function concept in secondary mathematics as a case study, we argue that representation conversion is not a technical skill superimposed upon conceptual understanding but is constitutive of mathematical thinking itself. We analyze how the function is expressed across five semiotic registers (natural language, algebraic-symbolic, graphical, tabular, and dynamic-technological) and identify four epistemological obstacles that emerge during conversion: the variable obstacle, the graph-as-picture obstacle, the formula-function conflation, and the correspondence-dependence tension. We further ground this analysis in specific examples drawn from the Chinese People's Education Press (PEP) textbook series and the 2022 Chinese mathematics curriculum standards, examining how curricular choices generate particular epistemological conditions. The paper contributes an integrative theoretical framework and a series of philosophical reflections on the nature of mathematical representation, concept formation, and the irreducibility of semiotic mediation to mathematical knowing.*

Keywords*— semiotic representation; epistemological obstacle; function concept; mathematics education; multiple representations

I. INTRODUCTION

1.1. The Paradox of Mathematical Access

Mathematics presents a unique epistemological situation among the disciplines. Unlike physics, where the objects of study (particles, fields, organisms) have some form of independent physical existence, mathematical objects (numbers, functions, geometric figures) are, in a strong sense, nothing apart from their representations. A number is not something one can see or touch; it is accessible only through numerals, words, gestures, or other semiotic means. A function is not a physical entity floating in space; it exists only insofar as it is expressed, denoted, or displayed through some system of signs.

Such a situation generates what we might call the paradox of mathematical access: mathematical objects are at once wholly certain (in a way that physical objects are not) and wholly inaccessible (in a way that physical objects are not). The function $y = 2x + 1$ is not the function itself; it is a representation of the function. The graph of $y = 2x + 1$ is not the function either; it is another representation. Yet we speak of "the function" as though it were a unified entity, a "same" object that persists across these different manifestations. What is this "same" that persists? And how does the learner come to grasp it?

These are not merely rhetorical questions. They are the foundational questions of a philosophy of mathematics education, and they frame the entire analysis that follows.

1.2. The State of the Literature

Mathematics education research has produced two largely parallel bodies of work relevant to these questions. The first concerns multiple representations in mathematics learning (Ainsworth, 2006; Arcavi, 2003; Goldin, 1998; Kaput, 1987). This literature examines how students work with different sign systems, including algebraic, graphical, verbal, and tabular, and how instruction might be designed to support their coordination. The second concerns epistemological obstacles in mathematics learning (Bachelard, 1938/2002; Brousseau, 1983, 1997; Sierpinska, 1985), which analyzes the conceptual barriers students encounter in their development of mathematical understanding.

Despite their clear relevance to one another, these two bodies of work have remained largely separate. Studies of multiple representations tend to treat conversion between registers as a cognitive skill, something that can be practiced and improved. Studies of epistemological obstacles tend to analyze conceptual difficulties without systematic attention to their semiotic dimensions. This separation, we argue, represents a significant theoretical gap.

1.3. Purpose and Approach

This paper addresses this gap by developing an integrated theoretical framework that brings together Duval's semiotic representation theory (Duval, 1995, 2006, 2017), Bachelard's epistemological obstacle concept (Bachelard, 1938/2002), and Sfard's reification framework (Sfard, 1991, 1992, 2008). We apply this framework to the function concept as it is developed in secondary mathematics, with particular attention to the Chinese curriculum context (Leung, 2006; Presmeg, 2006).

Our approach is purely theoretical. We do not present empirical data from classrooms or experiments. Instead, we offer a philosophical analysis, specifically a sustained reflection on the nature of mathematical representation, the obstacles that arise in learning, and the conditions under which genuine understanding becomes possible. This approach is appropriate for the questions we are asking, which are not empirical questions (what do students actually do?) but epistemological questions (what must happen for understanding to occur, and what prevents it?).

The paper proceeds as follows. Section 2 develops the theoretical framework, including philosophical reflections on the nature of representation itself. Section 3 constitutes the analytical core, examining the function concept across five registers and identifying four epistemological obstacles, grounded in specific examples from the Chinese curriculum. Section 4 offers a sustained philosophical discussion. Section 5 proposes pedagogical implications. Section 6 concludes.

II. THEORETICAL FRAMEWORK

2.1. What Is a Representation? A Philosophical Inquiry

Before we can analyze the multiple representations of mathematical concepts, we must ask a more fundamental question: What is a representation? The term is used so freely in mathematics education that its philosophical complexity is easily overlooked. Yet the concept of representation is not unproblematic.

In the Western philosophical tradition, representation has been understood in at least two distinct ways. The first, stemming from the empiricist tradition (Locke, Hume), treats representation as a kind of copy or mirror, in which the mind (or a sign) mirrors or copies something that exists independently. In this view, a graph "represents" a function in the same way that a photograph represents a field: there is an original, and the representation bears some structural resemblance to it.

The second tradition, stemming from Kant and developed through phenomenology and semiotics, treats representation not as a copy of a pre-existing reality but as a constitution of the object itself. In this view, the mathematical object does not exist "prior to" its representation; rather, it comes into being through the act of representation. Duval (1995) firmly aligns himself with this second tradition when he argues that mathematical objects are never given directly but only through and in their representations (Duval, 2006, p. 107). The representation does not stand behind the object; it is the only access to the object.

This is a strong claim, and its implications deserve careful unpacking. If mathematical objects are constituted through their representations, then:

(1) There is no "pure" mathematical object behind or beyond its representations. The function $y = 2x + 1$ is not a representation of something else; it is a presentation of the function. Different presentations (the graph, the table, the verbal description) are not different windows onto the same object but different modes of the object's being-given.

(2) Conversion between representations is not translation of a pre-existing content into different codes. It is a cognitive act that requires recognizing the same object in semiotically heterogeneous guises, an act that Duval (2006) calls noesis, the apprehension of a mathematical object.

(3) Different registers afford different aspects of the object. A graph shows global properties (shape, continuity) that an algebraic expression does not; an algebraic expression shows exact structure (coefficients, degree) that a graph does not. No single register exhausts the object.

This philosophical grounding is essential for the analysis that follows. It means that when we speak of "multiple representations," we are not speaking of multiple copies of the same thing but of multiple presentations that together constitute the mathematical object. The movement between them is not a mechanical translation but an epistemological achievement.

2.2. Duval's Theory of Semiotic Representation Registers

Duval's theory of semiotic representation registers (Duval, 1995, 2006, 2017) provides the primary architecture for our analysis. Duval distinguishes three fundamental cognitive processes involved in working with representations:

• Formation: The production of a representation within a single register (e.g., writing an expression, drawing a graph).

• Treatment: The transformation of a representation within the same register, governed by the register's internal rules (e.g., simplifying an expression, completing the square).

• Conversion: The transformation of a representation from one register to another while preserving the same mathematical object (e.g., going from an algebraic expression to its graph).

Duval's critical insight is that conversion is cognitively more demanding than treatment, and it is in conversion that the deepest difficulties of mathematical learning arise (Duval, 2006). The reason, in the terms we have developed above, is that conversion requires the learner to recognize the "same" mathematical object across semiotically heterogeneous representations, a recognition that cannot be accomplished algorithmically but demands a kind of conceptual flexibility that Duval links to genuine understanding.

Duval introduces the concept of noesis, the act of apprehending a mathematical object, and argues that noesis requires the coordination of at least two representation registers (Duval, 2006). A student who can only work with a function in its algebraic form has not truly grasped the function concept; understanding requires the ability to move between registers.

For the function concept in secondary mathematics, the relevant registers include:

• Natural language register: Verbal descriptions such as "y depends on x" or "for each input, there is exactly one output."

• Algebraic-symbolic register: Expressions such as $y = f(x)$, $y = 2x + 1$, or $f: A \to B$.

• Graphical register: The visual depiction of a function as a curve or set of points in a coordinate system.

• Tabular register: Tables of input-output values.

• Dynamic-technological register: Interactive representations in software such as GeoGebra or Desmos.

2.3. Epistemological Obstacles: Bachelard, Brousseau, Sierpinska

While Duval's framework illuminates the semiotic field, it does not explain why certain conversions are so persistently difficult. For this explanatory layer, we turn to Gaston Bachelard's concept of epistemological obstacle (Bachelard, 1938/2002).

Bachelard (1938/2002), in his philosophy of science, argued that scientific knowledge develops not by accumulation but by rupture, that is, by breaks with prior ways of thinking that constitute obstacles to new forms of knowing. In his formulation: the problem of scientific knowledge must be posed in terms of obstacles, and it is in the very act of knowing that one will identify the causes of stagnation and even of regression (Bachelard, 1938/2002, p. 14).

A crucial aspect of Bachelard's concept is that obstacles are not external impediments but arise from within cognition itself. They are the products of ways of thinking that were once productive and adequate but have become inadequate for new tasks. This differs fundamentally from the notion of a "misconception" or "error." A misconception can be corrected by providing the right information; an epistemological obstacle requires a restructuring of thought, a rupture with a prior mode of knowing.

Brousseau (1983, 1997) adapted Bachelard's concept to mathematics education, distinguishing three types of obstacles:

• Ontological obstacles: Arising from the nature of the mathematical object itself.

• Instrumental obstacles: Arising from the tools or instruments (including representations) used.

• Epistemological obstacles (strict sense): Arising from the very process of knowledge construction.

Sierpinska (1985) and Artigue (1991) extended this framework in their analyses of the limit concept, demonstrating that obstacles are not random but follow identifiable patterns reflecting the historical development of mathematical ideas.

2.4. Reification: Sfard's Process-Object Duality

Sfard's theory of reification (Sfard, 1991, 1992, 2008) provides a crucial bridging construct. Sfard argued that mathematical concepts have a dual nature: they can be conceived as processes (dynamic, operational) or as objects (static, structural). Mathematical thinking develops through three stages:

(1) Interiorization: Familiarity with operations.

(2) Compression: The ability to think of a sequence of operations as a unit.

(3) Reification: The sudden ability to see the compressed process as a static, manipulable object.

The function concept is a paradigmatic case. Initially, students encounter functions as computational processes (input → operation → output). Later, the function must be reified into an object, a thing that can be graphed, transformed, and composed. This reification is where many students stall (Breidenbach et al., 1992; Dubinsky, 1991; Sfard, 1992).

Critically, reification is deeply entangled with representation. Different semiotic registers emphasize different aspects of the process-object duality. An algebraic expression can be read operationally or structurally. A graph presents the function as a visual object but may obscure the operational aspect. The tabular register emphasizes discrete computation. Reification is thus not merely a cognitive event but a semiotic achievement; it requires the coordination of multiple registers.

2.5. The Integrative Framework

We propose the following integration:

Representation conversion difficulties are epistemological obstacles rooted in the process-object duality of mathematical concepts and the semiotic heterogeneity of their representations.

The framework posits a triangular relationship:

• Semiotic dimension (Duval): The multiple registers through which mathematical objects are presented

• Epistemological dimension (Bachelard/Brousseau): The obstacles arising from within knowledge construction

• Cognitive-developmental dimension (Sfard): The process-object duality and reification

Each dimension informs and constrains the others. The semiotic registers shape the obstacles; the obstacles reveal developmental stages; the stages determine which conversions are epistemologically feasible.

III. METHODOLOGY AND ANALYSIS

3.1. The Function Concept Across Five Semiotic Registers

We now examine how the function concept is expressed across five semiotic registers in secondary mathematics, analyzing what each register reveals, what it conceals, and, crucially, what this pattern of revelation and concealment tells us about the nature of mathematical objects themselves.

3.1.1. The Natural Language Register: The Primacy of Experience

In the natural language register, the function is typically described in terms of dependency and change. In the Chinese curriculum, this register is particularly prominent in the initial introduction of functions in Grade 8. The PEP textbook (People's Education Press, 2024, Grade 8, Volume 2, Chapter 22, §22.1, p. 90) introduces functions through a chapter introduction titled "Everything Changes" (万物皆变), noting that planetary positions change with time, the distance between China's Tiangong space station and the ground control center changes with time, temperature changes with altitude, and tree height changes with age, before presenting four concrete problems in its "Think" (思考) section:

• a car traveling at constant speed of 60 km/h, where distance $s = 60t$ as time t varies

• movie tickets cost 40 yuan each, where box office revenue $y = 40x$ as the number of tickets x varies

• circular ripples expand on water, where area $S = \pi r^2$ as radius r varies

• a rectangular prism with volume 1,000 cm^3, where height $h = 1000/S$ as base area S varies

These examples share a common linguistic pattern: one quantity changes as another changes (一个量随另一个量的变化而变化). This is the linguistic register of experience, involving the observation of phenomena in the world and noticing patterns of co-variation.

What this register reveals: The natural language register captures the intuitive, experiential dimension of function: the sense that quantities are related, that change is patterned, that one thing determines another. It provides the motivational grounding for the concept.

What this register conceals: The natural language descriptions carry implicit ontological assumptions that are rarely examined. When we say "the circumference changes as the radius changes," we implicitly posit a causal or temporal directionality: the radius causes the circumference to change, or the radius changes first and the circumference changes in response. But mathematically, this directionality is arbitrary; mathematically, the relationship is symmetric. The natural language register thus smuggles in an ontological framework (causality, temporality) that is foreign to the mathematical concept of function. This is not a trivial matter; it constitutes the first epistemological obstacle we will identify.

Philosophical reflection: This example illustrates a broader point about the natural language register. Language is never a neutral medium for expressing mathematical ideas; it carries within it a whole metaphysics, a way of carving up the world into subjects and predicates, causes and effects, before and after. When we use natural language to speak of mathematics, we are already importing a philosophical framework that may or may not be compatible with the mathematical objects we are trying to grasp. The task of mathematics education is, in part, the task of making students aware of this importation and helping them to achieve a degree of critical distance from it.

3.1.2. The Algebraic-Symbolic Register: Precision and Its Limits

The algebraic-symbolic register expresses the function through formal notation. In the PEP textbook (People's Education Press, 2024, §23.1), after introducing functions through natural language descriptions of co-variation, students are asked to identify the common structure of the examples and express it symbolically:

The textbook then introduces the linear function in symbolic form, stating that a function of the form $y = kx + b$, where k and b are constants with $k \neq 0$, is called a linear function. When $b = 0$, this reduces to $y = kx$, a proportional function, which is treated as a special case of the linear function (People's Education Press, 2024, Grade 8, Vol. 2, §23.1).

This symbolic formulation $y = kx + b$ (with the proportional function $y = kx$ as a special case) is a powerful compression: it captures in a few symbols what the natural language descriptions expressed through multiple examples. It allows algebraic manipulation (treatment), such as substitution, simplification, and composition, that would be unwieldy in natural language.

What this register reveals: Precision, compactness, generality. The notation $y = kx$ makes explicit the dependency relationship and allows one to derive properties (e.g., if $k > 0$, then y increases with x) through formal manipulation.

The textbook's phrasing, presenting a function of the form $y = kx + b$ as the definition, subtly suggests that the function is the form itself. This is the formula-function conflation obstacle.

Further reflection: The symbolic register raises a deeper question about the nature of mathematical objects. When we write $y = 2x + 1$, what exactly have we created? A formula is a syntactic object, a string of symbols arranged according to rules. A function is, in the modern conception, a set-theoretic entity, namely a mapping between sets. How can these be "the same"? The answer, of course, is that they are not the same; the formula represents the function. But the symbolic register, by its very nature, tends to efface this distinction. The formula becomes so closely associated with the function that the difference between representation and represented vanishes from awareness. This goes beyond pedagogy; it is a structural feature of the symbolic register that has deep roots in the philosophy of mathematics (Carlson et al., 2001; Duval, 2006).

3.1.3. The Graphical Register: The Global View and Its Illusions

The graphical register depicts the function as a visual object, specifically a line or curve in a coordinate plane. The PEP textbook (People's Education Press, 2024, §23.2) introduces graphing of proportional functions, where students are asked to plot points for $y = 2x$ and connect them to form a line through the origin. Research on visual thinking in mathematics (Arcavi, 2003) highlights both the power and the pitfalls of graphical reasoning.

Affordances: The graphical register provides a global, simultaneous view of the function. The entire relationship between input and output is visible at once. It affords visual reasoning about properties such as monotonicity (is the line going up or down?), linearity (is it straight?), and behavior at extreme values (Arcavi, 2003).

Limitations: The graph presents a continuous, apparently complete object, but this is an illusion. The graph on paper (or screen) shows only a finite window of the function. The line "extends to infinity" in both directions, but this extension is not shown; it is imagined. Moreover, the graph can promote the misconception that a function must be a continuous curve, or that the graph is a physical "thing" rather than a mathematical representation (Goldin, 1998).

Broader implications: The graphical register introduces a fascinating tension between the visible and the invisible. The graph shows us something (a line, a curve), but what it shows is only a trace of the function, a finite excerpt from an infinite totality. The mathematical function extends beyond what the graph can display. This means that reading a graph always involves an act of transcendence, going beyond what is given to what is implied. This act of transcendence is itself an epistemological achievement, one that students often fail to make (Arcavi, 2003; Duval, 2006). When a student looks at a graph of a proportional function and sees "a line going up," they are seeing the trace but not the function. The function is not the line; it is the law that the line traces.

3.1.4. The Tabular Register: Discreteness and Computation

The tabular register presents the function as a set of discrete input-output pairs. In the PEP textbook (People's Education Press, 2024), tables are frequently used in the initial exploration of functions. For example, when introducing proportional functions, the textbook presents tables of input-output pairs for students to observe patterns.

Affordances: The tabular register makes the function concrete and computational. It emphasizes the operational aspect: specific inputs produce specific outputs. It connects directly to the concept of function as a "machine" or "rule" (Sfard, 1991, 1992).

Limitations: The table shows only a finite sample. The gap between the discrete table and the continuous function is a significant epistemological gap. Moreover, the tabular register tends to reinforce the process-conception of function (as a computation) rather than the object-conception (as a unified entity) (Dubinsky, 1991; Sfard, 1991).

Epistemological note: The tabular register is perhaps the most "empirical" of all the registers: it presents the function as a collection of observed data points, akin to measurements in a laboratory. This is both its strength (it connects mathematics to experience) and its limitation (it cannot capture the mathematical generality of the function). The movement from a finite table to the general function is an act of mathematical induction, an epistemological leap from the particular to the universal. This leap is not logically guaranteed (a finite number of data points cannot determine a function uniquely), yet it is essential to mathematical thinking (Artigue, 1991).

3.1.5. The Dynamic-Technological Register: Interactivity and Its Epistemology

Dynamic software environments such as GeoGebra introduce a new semiotic register: the dynamic-technological register. The 2022 Chinese curriculum standards (Ministry of Education, P. R. China, 2022) explicitly encourage the use of technology, noting that information technology can provide students with rich learning resources and powerful tools for mathematical exploration (Ministry of Education, P. R. China, 2022, p. 53). Studies of technology-enhanced mathematics learning (Kaput et al., 2008; Radford, 2003) suggest that dynamic environments afford unique forms of mathematical exploration.

Affordances: The dynamic register affords a kind of experimental mathematics: students can explore how changes in parameters affect the function, observe patterns of covariation in real time, and develop intuitions about properties through direct interaction (Radford, 2003).

Limitations: The dynamic register can create the illusion that mathematical truth is established by empirical observation rather than proof. It can also obscure the mathematical structure behind the visual display (Kaput et al., 2008).

Epistemological note: The dynamic register raises questions about the relationship between mathematics and technology. When a student drags a slider in GeoGebra and watches a graph change in real time, what kind of mathematical activity is this? Is it experiment (like a physics experiment)? Is it exploration? Or is it something new, a hybrid form of mathematical activity that blends the deductive certainty of mathematics with the empirical contingency of technology? This extends beyond pedagogy; it touches on the nature of mathematical knowing itself (Lesh et al., 1987).

3.2. Epistemological Obstacles in Representation Conversion

We now identify four epistemological obstacles that emerge when students attempt to convert between the representation registers described above. Each obstacle represents a point at which prior ways of thinking, supported by specific registers, become inadequate for the demands of cross-register conversion.

3.2.1. The Variable Obstacle: From Experience to Abstraction

In the natural language register, students encounter the idea that one quantity changes as another changes. The variable is understood as a changing quantity: time passes, temperature drops, a spring stretches. This is the register of the PEP textbook's introductory examples, where the distance s changes as time t changes (People's Education Press, 2024, p. 90).

When students move to the algebraic-symbolic register, however, the variable takes on a different status. In $y = kx$, the letter x is not simply "a changing quantity"; rather, it is simultaneously a placeholder for any value in the domain, an unknown that can be solved for, and a structural element in a relational expression. These are different modes of being for the variable, and they are not naturally compatible (Carlson et al., 2001; Sfard, 1991).

Analysis of the obstacle: This is an ontological obstacle in Brousseau's sense (Brousseau, 1983, 1997). The variable as a mathematical object has a different ontological status from the variable as an experienced quantity. The experienced variable is temporal, concrete, particular. The mathematical variable is atemporal, abstract, general. The movement from one to the other requires a rupture with the ontology of experience, a rupture of the kind Bachelard (1938/2002) identified as central to scientific knowledge formation.

Deeper analysis: The variable obstacle reveals something fundamental about the nature of mathematical abstraction. When we write $y = 2x$, we are not describing a particular instance of change; we are expressing a law that governs all possible instances simultaneously (Artigue, 1991; Sierpinska, 1985). The letter x does not stand for this value or that value; it stands for any value, and for all values at once. This is an extraordinary mode of thought, one that has no parallel in everyday experience. It is, in Bachelard's terms, an epistemological act: the constitution of a new kind of object that transcends the given.

3.2.2. The Graph-as-Picture Obstacle: From Perception to Mathematics

When students convert from the algebraic register ($y = 2x + 1$) to the graphical register (a line in the coordinate plane), they frequently interpret the graph as a "picture" of a physical situation rather than as a mathematical representation of an abstract relationship. A graph of temperature versus time is read as "a picture of the weather getting colder" rather than as a depiction of the functional relationship between two abstract quantities (Arcavi, 2003).

Nature of the obstacle: This is an instrumental obstacle (Brousseau, 1983, 1997). The graph, as an instrument of representation, is misused because students apply perceptual rather than mathematical reasoning to it. The graph-as-picture interpretation is natural, as it draws on well-developed perceptual capacities, but it is epistemologically inadequate for mathematical purposes.

Deeper analysis: The graph-as-picture obstacle illustrates a tension that runs throughout the philosophy of mathematics: the relationship between the sensible and the intelligible. A graph is a sensible object: it can be seen, touched, measured. But the function it represents is an intelligible object: it cannot be perceived but only thought. The act of reading a graph mathematically requires a kind of epoche (in the phenomenological sense), namely a suspension of the natural attitude toward the graph as a physical object, in order to apprehend it as a presentation of a mathematical entity. This epoche is not trivial; it is a significant epistemological achievement (Duval, 1995, 2006).

3.2.3. The Formula-Function Conflation: The Seduction of Notation

Students who have primarily encountered functions through algebraic expressions may come to believe that the expression is the function. This is perhaps the most pervasive and insidious of all the obstacles (Sfard, 1991, 1992).

Nature of the obstacle: This is an epistemological obstacle in the strict Bachelardian sense (Bachelard, 1938/2002). The identification of function with formula was once productive, as it enabled algebraic manipulation and computational competence, but it now becomes an obstacle to recognizing the function as an abstract object that transcends any particular representation. Research on concept images (Tall & Vinner, 1981) shows that learners' mental representations of mathematical concepts often diverge significantly from the formal definitions, and this divergence is particularly acute in the case of the function concept.

Deeper analysis: The formula-function conflation is not merely a student error; it reflects a deep-seated tendency in mathematical practice itself. Mathematicians routinely speak of "the function $y = 2x + 1$" as though the function and its expression were identical. This is a kind of metonymy: the part standing for the whole, the representation being identified with the represented. Such metonymy is pragmatically useful but philosophically problematic. It obscures the fundamental question: What is the relationship between the symbolic expression and the mathematical object? The answer, from our framework, is that the relationship is one of presentation (Darstellung), not identity. The formula presents the function; it is not the function (Duval, 2006).

3.2.4. The Correspondence-Dependence Tension: A Historical Recapitulation

The fourth obstacle arises from the tension between two conceptions of function: the dependency view (函数是变量之间的依赖关系) and the correspondence view (函数是两个集合之间的对应关系).

In the PEP textbook (People's Education Press, 2024), the dependency view dominates the Grade 8 introduction. Students encounter functions through situations of co-variation, where one quantity changes as another changes. It is only in high school that the formal correspondence definition is introduced, as specified in the curriculum standards (Ministry of Education, P. R. China, 2020): a function is defined as a mapping from set A to set B such that each element of A is assigned exactly one element of B (People's Education Press, 2019, §3.1).

Nature of the obstacle: This tension is an epistemological obstacle (Bachelard, 1938/2002; Brousseau, 1983, 1997; Sierpinska, 1985) that mirrors the historical development of the function concept itself. The evolution from Euler's and Leibniz's analytic definition (function as an expression) through Dirichlet's dependency definition to the modern set-theoretic correspondence definition represents the kind of epistemological rupture that Bachelard (1938/2002) identified. Students, in their learning trajectory, recapitulate this history (Artigue, 1991; Carlson et al., 2001; Sierpinska, 1985).

Deeper analysis: The correspondence-dependence tension reveals something significant about the relationship between the history of mathematics and the learning of mathematics. The famous "recapitulation thesis" (ontogeny recapitates phylogeny) has been much debated in mathematics education, but our analysis suggests that it has a kernel of truth at the level of epistemological obstacles. The difficulties students encounter are not arbitrary; they mirror the difficulties that the mathematical community historically encountered. This is not because students are "repeating" history in any literal sense, but because the conceptual terrain they must traverse is structured by the same epistemological constraints that structured the historical development (Sierpinska, 1985). The dependency view is not "wrong"; it was the state of mathematical understanding for centuries, but it is inadequate for the demands of modern mathematics. Recognizing this inadequacy and moving beyond it is an epistemological achievement of the first order.

IV. DISCUSSION

4.1. Representation Conversion as Ontological Rupture

The central claim of this paper is that representation conversion is not a secondary skill but a constitutive act of mathematical thinking. This claim has far-reaching philosophical implications that deserve sustained reflection.

When a student converts from the natural language register ("y changes as x changes") to the algebraic register ($y = 2x$), what happens is not merely a change in notation. It is an ontological transformation: a shift from the ontology of experience (where quantities change in time, where causality reigns, where particular instances are given) to the ontology of mathematics (where relationships are atemporal, where all instances are co-present, where the particular is transcended in favor of the universal). This transformation is what Bachelard (1938/2002) called an epistemological rupture, a break with the immediacy of experience in favor of a mediated, constructed understanding.

The same can be said for each of the conversions we have analyzed. Converting from formula to graph is not merely "drawing a picture"; rather, it is an act of spatialization, of making visible a relationship that was previously only expressed symbolically. Converting from graph to table is an act of discretization, of extracting particular instances from a continuous whole. Each conversion is an epistemological act, a reconstitution of the mathematical object in a new register, with new affordances and new constraints (Duval, 1995, 2006).

4.2. The Irreducibility of Semiotic Mediation

A deeper philosophical point emerges from our analysis: semiotic mediation is irreducible to any non-semiotic substrate. There is no "pure" mathematical object behind or beyond its representations. The function is not something that exists independently and then gets "clothed" in various representations. The function is its presentations; it exists only in and through them.

Such a claim may seem counterintuitive. We speak of "the function $y = 2x + 1$" as though there were a single entity that persists across different representations. But what is this "single entity"? It is not the formula (that is one representation). It is not the graph (that is another). It is not the table (another still). If we subtract all the representations, what is left? Nothing; there is no pure, unrepresented "function-in-itself."

This is not to say that the function is merely its representations. The various representations are not arbitrary or interchangeable; they are constrained by the mathematical object they present. A correct graph of y =

2x + 1 is not just any old line; it is this particular line, and it would be wrong to draw a parabola. The mathematical object constrains its presentations, even though it has no existence apart from them (Duval, 2006, 2017).

This position, sometimes called structural realism in the philosophy of mathematics, has important implications for mathematics education. It means that we cannot teach students to "understand the function" by first giving them the "pure concept" and then showing them how it gets represented. There is no "pure concept" accessible apart from representations. Understanding the function is the ability to navigate its representations, to move between them, to translate, to compare, to synthesize (Ainsworth, 2006; Duval, 1995; Kaput, 1987).

4.3. The Historical-Epistemological Dimension

Our analysis has revealed a striking parallel between the epistemological obstacles students encounter and the historical development of the function concept. The dependency view (which students first encounter) corresponds to the pre-modern conception of function (Leibniz, Euler). The correspondence view (which students encounter later) corresponds to the modern set-theoretic conception (Dirichlet, Bourbaki).

The parallel is not coincidental. It reflects what we might call the epistemological topology of the function concept, the field of conceptual possibilities and constraints that any thinker (individual or historical) must navigate. The terrain is the same whether one is a 17th-century mathematician or a 21st-century student; what differs is the starting position and the resources available for navigation (Artigue, 1991; Bachelard, 1938/2002; Sierpinska, 1985).

This observation has a profound implication: the difficulties students encounter are not signs of individual failure but are structural features of the conceptual terrain itself. No student, regardless of intelligence or effort, can avoid these difficulties; they are built into the nature of the concept. The task of education is not to eliminate these difficulties (which is impossible) but to make them visible and navigable, so as to help students recognize them as obstacles rather than personal failings, and to provide them with the resources to move through and beyond them (Brousseau, 1983, 1997).

4.4. The Chinese Curriculum as an Epistemological Site

Our analysis of the Chinese curriculum context reveals something important: curriculum design is never epistemologically neutral. The choices made about which registers to emphasize, in what order, and in what contexts, have epistemological consequences that shape the obstacles students will encounter (Ministry of Education, P. R. China, 2020, 2022; People's Education Press, 2024).

The Chinese curriculum's emphasis on variation (变化) and co-variation provides rich motivational grounding but also strengthens the dependency conception, making the transition to the correspondence view more difficult (Ministry of Education, P. R. China, 2022; People's Education Press, 2024). The emphasis on symbolic manipulation (the "two basics" or 双基 tradition) ensures computational fluency but may strengthen the formula-function conflation (Schneider, 2014). The tradition of variation pedagogy (变式教学) has significant potential for supporting representation conversion, but only when the variation is directed at the semiotic-epistemological dimensions, not merely at procedural fluency (Ma, 1999).

These observations have implications beyond the Chinese context. They suggest that every curriculum creates a specific "epistemological profile", a configuration of obstacles and affordances, that shapes the conceptual journey students must undertake. Understanding this configuration, making its contours visible, is a crucial task for mathematics education research (Presmeg, 2006).

The PEP textbook’s 2024 edition (People’s Education Press, 2024; approved for classroom use beginning Spring 2026) represents a significant restructuring. The function concept is now introduced in Chapter 22 (“函数”) through the chapter theme “Everything Changes” (万物皆变), using four diverse examples, including uniform motion ($s = 60t$), box office revenue ($y = 40x$), circular area ($S = \pi r^2$), and inverse variation ($h = 1000/S$), that deliberately include proportional, quadratic, and inverse relationships, not only proportional ones. Linear functions and proportional functions are then treated in Chapter 23 (“一次函数”), where the general linear function $y = kx + b$ is introduced first, with the proportional function $y = kx$ treated as the special case where $b = 0$. This “general to specific” approach differs from the prior edition’s strategy of introducing proportional functions first. Investigating the epistemological implications of this curricular restructuring, particularly how the choice of introductory examples and the sequencing of general-before-specific shapes students’ initial apprehension of the function concept, would be a valuable direction for future research.

4.5. Limitations and Future Directions

This study is a theoretical analysis and does not present empirical data. The obstacles we have identified are derived from philosophical reasoning and curriculum analysis rather than from systematic observation of student learning. While our analysis is consistent with the existing empirical literature (Ainsworth, 2006; Goldin, 1998; Kaput, 1987; Sfard, 1991, 1992; Tall & Vinner, 1981), the specific claims about epistemological obstacles would benefit from empirical validation through classroom-based research.

Additionally, while we have focused on the function concept as a paradigmatic case, the applicability of our framework to other mathematical concepts (limit, derivative, integral, probability) has not been explored here and represents an important direction for future research (Artigue, 1991; Sierpinska, 1985).

A further limitation is that our philosophical reflections, while drawing on the Western philosophical tradition (Kant, phenomenology, structural realism), have not engaged deeply with Chinese philosophical traditions or with the specific epistemological assumptions embedded in Chinese mathematics education. This engagement would be a valuable extension of the present work (Leung, 2006; Presmeg, 2006; Schneider, 2014).

V. PEDAGOGICAL IMPLICATIONS

Drawing on our theoretical analysis, we propose five pedagogical implications grounded in epistemological awareness.

5.1. Make the Ontological Shift Explicit

Teachers should make explicit to students that the move from "the circumference changes as the radius changes" to "y = kx" is not merely a change in notation but a fundamental transformation in the kind of object being considered. Students should be helped to see that they are moving from the world of experience (where things change in time) to the world of mathematics (where relationships are atemporal and general). This is not a small step; it is an epistemological rupture in Bachelard's (1938/2002) sense, and it should be treated as such in instruction.

5.2. Teach Representation Conversion as Epistemological Work

Representation conversion should not be treated as a routine skill to be practiced. Drawing on Duval's (1995, 2006, 2017) distinction between treatment and conversion, teachers should recognize that conversion demands something qualitatively different from treatment: not the application of rules, but the recognition of a mathematical object across semiotically heterogeneous presentations (Duval, 2006). Teachers should explicitly discuss what each register reveals and conceals, and should design tasks that specifically target the epistemological dimensions of conversion.

5.3. Use Errors as Diagnostic of Epistemological Stage

Students' errors in representation conversion should be interpreted not as failures but as indicators of their current epistemological stage. A student who treats a graph as a "picture" is not making a trivial mistake; they are revealing a mode of thought that needs to be refined, not simply corrected. The concept of epistemological obstacle (Bachelard, 1938/2002; Brousseau, 1983, 1997; Sierpinska, 1985) provides a framework for understanding the productive logic of such errors, since errors are not random but reflect the learner's current position in the conceptual terrain. Tall and Vinner's (1981) concept of concept image versus concept definition is similarly relevant here: the gap between a student's mental representation of a function and the formal definition is not a deficit but a developmental stage.

5.4. Design "Bridging Situations"

We propose the concept of bridging situations, defined as instructional situations specifically designed to target the epistemological obstacles that arise during representation conversion. Drawing on Brousseau's (1983, 1997) theory of didactical situations, a bridging situation presents a task that cannot be solved within a single register but requires movement between registers, forcing the epistemological work of conversion. For example: "Find a function whose graph passes through the origin, whose algebraic expression has no constant term, and whose table of values shows equal increments." Such tasks require the coordination of graphical, algebraic, and tabular registers simultaneously, demanding the kind of noetic act that Duval (2006) identifies as essential for mathematical comprehension.

5.5. Leverage Variation Pedagogy for Epistemological Ends

The Chinese tradition of variation pedagogy (变式教学) has significant potential for supporting epistemological development (Ma, 1999). Teachers should use systematic variation not merely to vary procedural contexts but to vary the semiotic-epistemological dimensions of the concept, presenting the same function in different registers, asking students to compare and convert, making the process of representation conversion visible and explicit. This approach aligns with the broader goals of the 2022 curriculum standards (Ministry of Education, P. R. China, 2022), which emphasize core mathematical competencies（核心素养）including mathematical abstraction, logical reasoning, and mathematical modeling.

VI. CONCLUSION

This paper has presented a sustained theoretical analysis of the epistemological dimensions of multiple representations in secondary mathematics, using the function concept as a case study. Our central argument is that the difficulties students experience when converting between different representations of a mathematical concept are not merely technical or cognitive obstacles; they are epistemological obstacles that arise from within the process of mathematical knowing itself.

We have developed an integrative framework that brings together three theoretical perspectives: Duval's semiotic representation theory (Duval, 1995, 2006, 2017), Bachelard's epistemological obstacle concept (Bachelard, 1938/2002), and Sfard's reification framework (Sfard, 1991, 1992, 2008). Through this framework, we have identified four epistemological obstacles in the conversion of function representations: the variable obstacle, the graph-as-picture obstacle, the formula-function conflation, and the correspondence-dependence tension. We have grounded this analysis in specific examples from the Chinese PEP textbook (People's Education Press, 2024) and curriculum standards (Ministry of Education, P. R. China, 2020, 2022).

Our analysis contributes several philosophical reflections that extend beyond the specific case of the function concept:

1. Representation is constitution, not copy: Mathematical objects are constituted through their representations, not merely represented by them (Duval, 1995, 2006).

2. Conversion is rupture, not translation: The movement between registers involves an epistemological break with prior modes of thought, not a mechanical transformation (Bachelard, 1938/2002; Duval, 2006).

3. Obstacles are structural, not accidental: The difficulties students encounter are built into the conceptual terrain of mathematics itself, not merely the result of poor instruction or individual deficiency (Bachelard, 1938/2002; Brousseau, 1983, 1997; Sierpinska, 1985).

4. Curriculum is epistemologically loaded: The choices made in curriculum design have epistemological consequences that shape the obstacles and affordances of the learning journey (Brousseau, 1983, 1997; Ministry of Education, P. R. China, 2022; People's Education Press, 2024).

These reflections have implications for both theory and practice in mathematics education. They suggest that the task of mathematics education is not merely to transmit mathematical content but to support students in the demanding epistemological work of constructing mathematical objects through, across, and beyond their representations.

REFERENCES

[1] Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*(3), 183–198. https://doi.org/10.1016/j.learninstruc.2006.03.001

[2] Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241. https://doi.org/10.1023/A:1024312321077

[3] Artigue, M. (1991). Épistémologie et didactique. *Recherches en Didactique des Mathématiques, 10*(2–3), 241–286.

[4] Bachelard, G. (2002). *The formation of the scientific mind: A contribution to a psychoanalysis of objective knowledge* (M. McAllester, Trans.). Clinamen Press. (Original work published 1938)

[5] Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). *Development of the process conception of function*. In Proceedings of the 16th Conference of the International Group for the Psychology of Mathematics Education (Vol. 2, pp. 25–32). PME.

[6] Brousseau, G. (1983). Les obstacles épistémologiques et les problèmes en mathématiques. *Recherches en Didactique des Mathématiques, 4*(2), 165–198.

[7] Brousseau, G. (1997). *Theory of didactical situations in mathematics* (N. Cooper & R. Sutherland, Trans.). Kluwer Academic Publishers.

[8] Carlson, M., Larsen, S., & Jacobs, S. (2001). *An investigation of covariational reasoning and its role in learning the concepts of limit and accumulation*. In Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education – North America (Vol. 2, pp. 145–149). PME-NA.

[9] Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 95–123). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47209-0_6

[10] Duval, R. (1995). *Sémiosis et pensée humaine: Registres sémiotiques et apprentissages intellectuels*. Peter Lang.
[11] Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1–2), 103–131. https://doi.org/10.1007/s10649-006-0400-z
[12] Duval, R. (2017). *Understanding the mathematical way of thinking: The registers of semiotic representations* (C. E. Morales Vega, Trans.). Springer. https://doi.org/10.1007/978-3-319-56910-9
[13] Goldin, G. A. (1998). Representational systems, learning, and problem solving in mathematics. *The Journal of Mathematical Behavior, 17*(2), 137–165. https://doi.org/10.1016/S0364-0213(99)80056-1
[14] Kaput, J. (1987). Towards a language theory for the study of mathematics education. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 3–16). Lawrence Erlbaum Associates.
[15] Kaput, J. J., Blanton, M. L., & Moreno-Armella, L. (2008). *Algebra from a symbolization point of view*. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), Algebra in the elementary grades: Research and developmental perspectives (pp. 34–63). Routledge.
[16] Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33–40). Lawrence Erlbaum Associates.
[17] Leung, F. K. S. (2006). *Mathematics education in China: Tradition and change*. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education (Vol. 1, pp. 17–44). PME.
[18] Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Lawrence Erlbaum Associates. https://doi.org/10.4324/9781410602589
[19] Ministry of Education, P. R. China. (2020). Senior high school mathematics curriculum standards (2017 ed., revised 2020) [普通高中数学课程标准（2017年版 2020 年修订）]. People's Education Press.
[20] Ministry of Education, P. R. China. (2022). Compulsory education mathematics curriculum standards (2022 ed.) [义务教育数学课程标准（2022 年版）]. Beijing Normal University Press.
[21] People's Education Press. (2019). Mathematics (Compulsory Volume 1) [数学（必修第一册）]. (人教 A 版 2019).
[22] People's Education Press. (2024). Mathematics (Grade 8, Volume 2) [数学（八年级下册）]. (人教版 2024).
[23] Presmeg, N. (2006). Sociocultural lenses for research on visualization in mathematics education. *Educational Studies in Mathematics, 61*(1–2), 1–8. https://doi.org/10.1007/s10649-006-0401-y
[24] Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generationalizations. *Mathematical Thinking and Learning, 5*(1), 1–36. https://doi.org/10.1207/s15327833mtl0501_02
[25] Schneider, M. (2014). Epistemological obstacles in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 315–320). Springer. https://doi.org/10.1007/978-94-007-4978-8_57
[26] Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22*(1), 1–36. https://doi.org/10.1007/BF00302715
[27] Sfard, A. (1992). The development of reification: From process to object. *For the Learning of Mathematics, 12*(3), 15–21.
[28] Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press. https://doi.org/10.1017/CBO9780511499944
[29] Sierpinska, A. (1985). Obstacles épistémologiques relatifs à la notion de limite. *Recherches en Didactique des Mathématiques, 6*(1), 5–68.
[30] Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151–169. https://doi.org/10.1007/BF00305619